Correlations and energy transfer in compressible isothermal and adiabatic MHD turbulence

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in collaboration with
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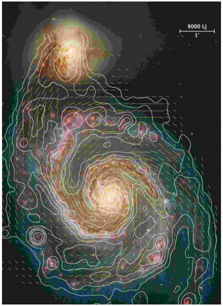
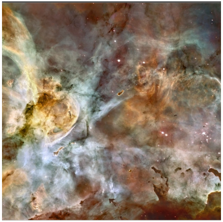
Motivation

- In many astrophysical systems
  - turbulence
  - compressibility
  - magnetic fields

  e.g. dynamos, accretion disks, cosmic rays, star formation
- How do they interact?
- How can we model these processes?

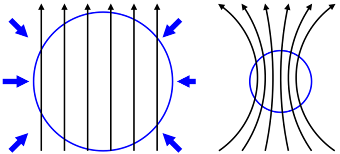
⇒ Study energy transfer



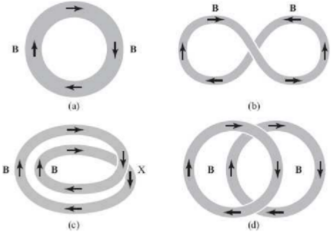


[Image credit top: MPIfR and Newcastle University]

Magnetohydrodynamic (turbulence)

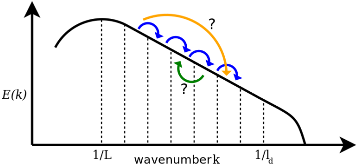


- "Frozen-in" magnetic field lines
- Rich interaction between kinetic and magnetic energy budgets
- Magnetic tension
- Magnetic pressure



[Dynamo image credit: Vainshtein & Zel'dovich '72]

- Energy transfers
  - Energy cascade
  - Inverse transfer
  - Nonlocal transfer



Energy budgets in incompressible MHD [e.g., Verma 2004, Alexakis+ 2005]

$$E_u(K) = \sum_Q \int - \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{u} \cdot \nabla)\, \mathbf{w}^{\mathrm{Q}}}_{\text{advection (kinetic cascade)}}$$

$$+ \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{v}_{\mathrm{A}} \cdot \nabla)\, \mathbf{B}^{\mathrm{Q}}}_{\text{magnetic tension}} \qquad + \cdots \mathrm{d}\mathbf{x}$$

$$E_b(K) = \sum_Q \int - \underbrace{\mathbf{B}^{\mathrm{K}} \cdot (\mathbf{u} \cdot \nabla)\, \mathbf{B}^{\mathrm{Q}}}_{\text{advection (magnetic cascade)}}$$

$$+ \underbrace{\mathbf{B}^{\mathrm{K}} \cdot \nabla \cdot \left(\mathbf{v}_{\mathrm{A}} \otimes \mathbf{w}^{\mathrm{Q}}\right)}_{\text{magnetic tension}} \qquad + \cdots \mathrm{d}\mathbf{x}$$

Energy budgets in compressible MHD

[Grete+ PoP 2017]

$$E_u(K) = \sum_Q \int - \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{u} \cdot \nabla)\, \mathbf{w}^{\mathrm{Q}}}_{\text{advection (kinetic cascade)}} - \underbrace{\frac{1}{2} \mathbf{w}^{\mathrm{K}} \cdot \mathbf{w}^{\mathrm{Q}} \nabla \cdot \mathbf{u}}_{\text{compression}}$$

$$+ \underbrace{\mathbf{w}^{\mathrm{K}} \cdot (\mathbf{v}_{\mathrm{A}} \cdot \nabla)\, \mathbf{B}^{\mathrm{Q}}}_{\text{magnetic tension}} - \underbrace{\frac{\mathbf{w}^{\mathrm{K}}}{2\sqrt{\rho}} \cdot \nabla \left(\mathbf{B} \cdot \mathbf{B}^{\mathrm{Q}} \right)}_{\text{magnetic pressure}} + \cdots \mathrm{d}\mathbf{x}$$

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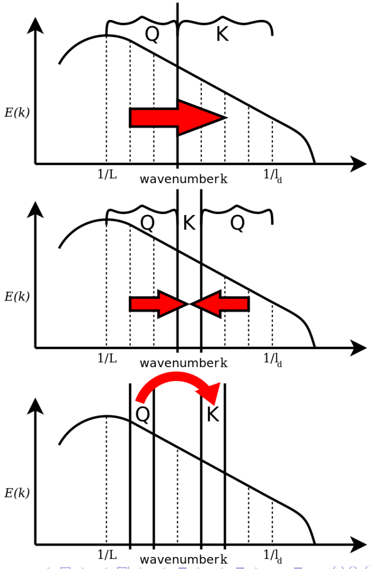
Eddies in motion

[Grete+ PoP 2017]

- Driven sub- and supersonic MHD turbulence

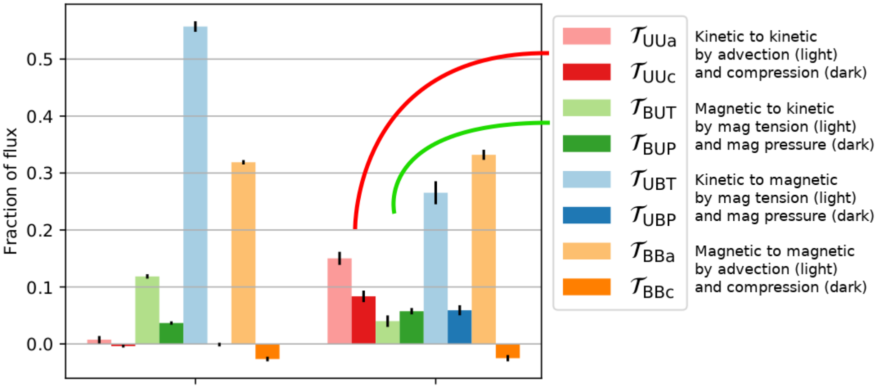
What can we learn from the transfer functions $\mathcal{T}_{\mathrm{XYZ}}(Q,K)$?

- Cross-scale transfer: $\sum_{Q\leq k}\sum_{K>k}\mathcal{T}$
  e.g. relevant for subgrid-scale turbulence modeling
- Total transfer: $\sum_{Q}\mathcal{T}$
  e.g. relevant for the net effects cf. dynamos
- Shell-to-shell transfer: $\mathcal{T}$
  helps to explain ~~everything~~ a lot



Mean cross-scale flux in the inertial range

[Grete+ PoP 2017]



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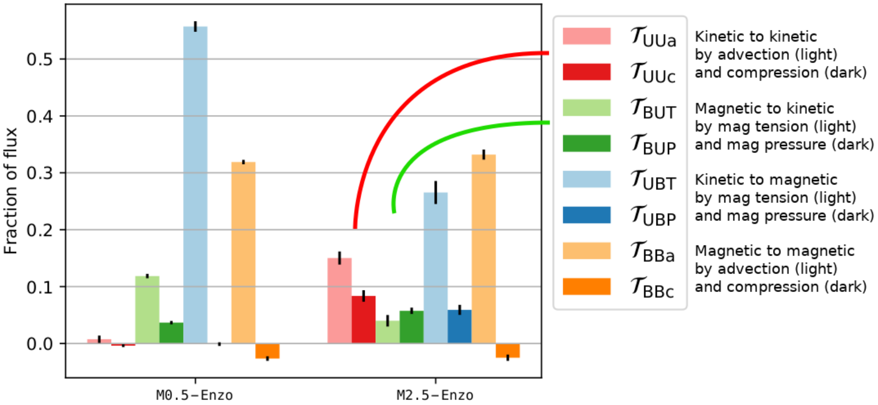
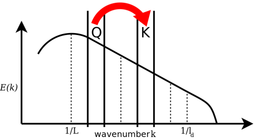


- Subsonic transfers match results of spectral code [Debliquy+ PoP 2011]
- Supersonic transfers are more dynamic

The energy cascades

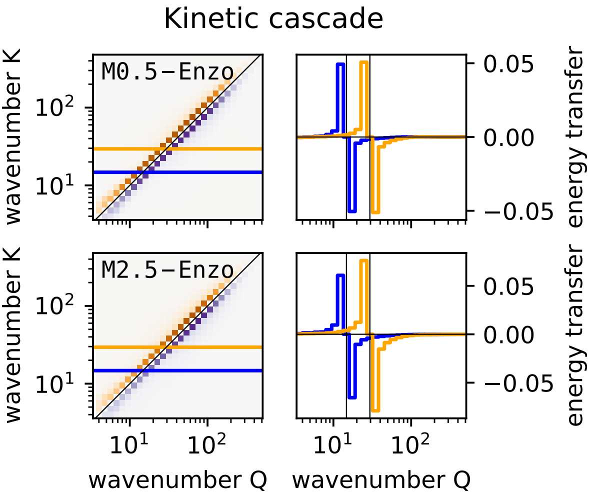
[Grete+ PoP 2017]



- Energy transfer is local

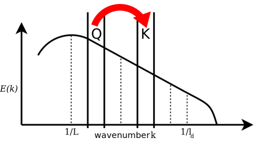
⇒ Shell N
  - receives energy from shell $N-1$
  - transfer energy to shell $N+1$

- Applies to (the stronger) magnetic cascade, too

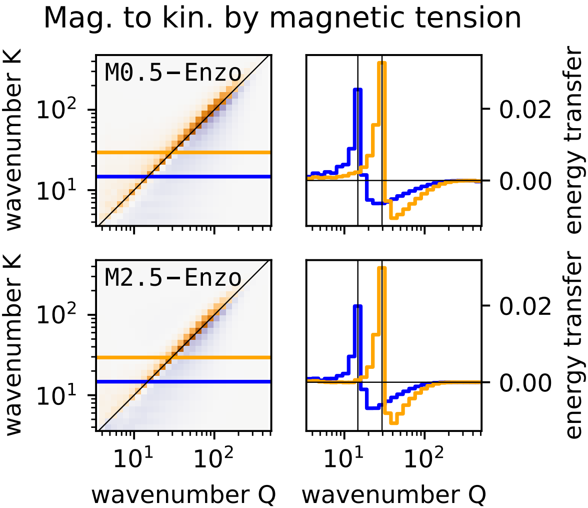


Transfer mediated by magnetic tension

[Grete+ PoP 2017]



- Energy transfer is weakly local
- Velocity and magnetic field exchange most energy at $K = Q$
- Energy is received from few larger scales $Q \leq K$ and transferred to more smaller scales $Q > K$



Correlations in adiabatic turbulence

[Grete+ in prep.]

- Subsonic, super-Alfvénic
- Influence of thermodynamics
  - $\gamma = 1.0001 \sim$ isothermal
  - $\gamma = 7/5 \sim$ diatomic gas
  - $\gamma = 5/3 \sim$ monoatomic gas
- Cooling
  - $\mathcal{L} \propto \rho T \sim$ linear
  - $\mathcal{L} \propto \rho^2 \sqrt{T} \sim$ free-free

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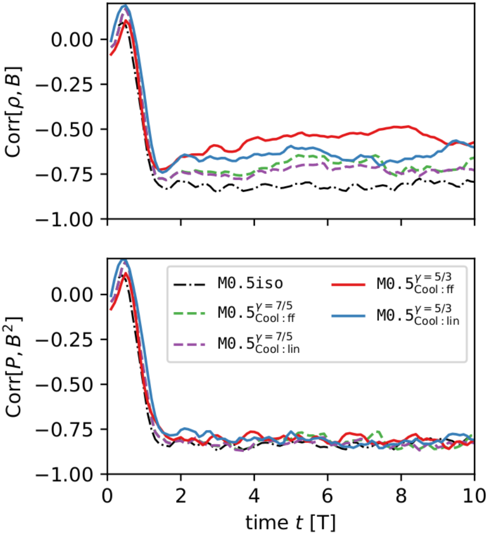
- ρ-B sensitive to EOS

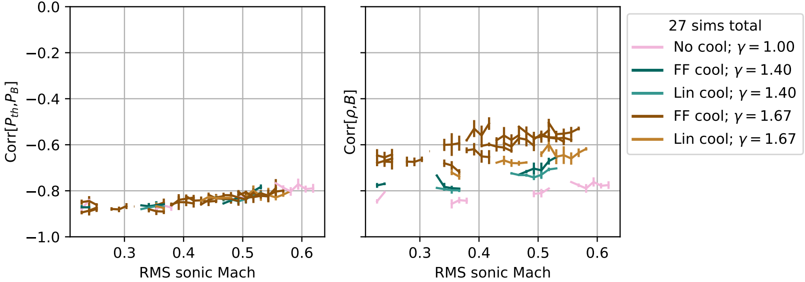
⇒ important(?) to Faraday rotation

- P-B^2 unaffected by EOS

⇒ total pressure equil./slow mode



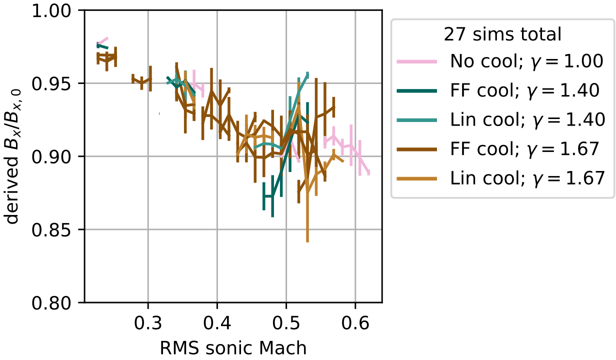
Correlations in adiabatic turbulence at varying $\mathrm{M_s}$ [Grete+ in prep.]



- (strong anti-)correlation between pressures remains
- $\rho - B$ (anti-)correlation is sensitive to EOS across $\mathrm{M_s}$

Derived magnetic field measurements

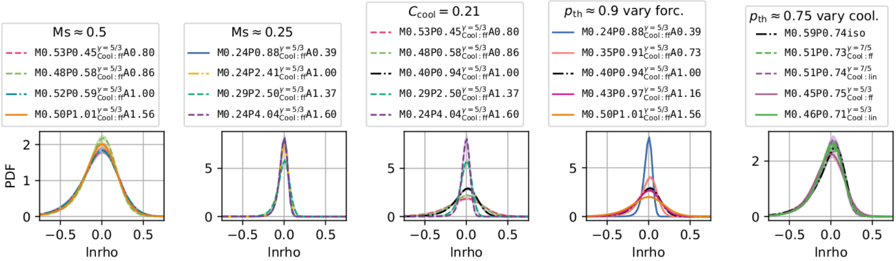
[Grete+ in prep.]



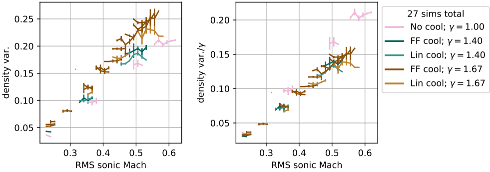
- Varying $\mathrm{M_s}$ more important than EOS
- Question: Is this relevant?

Density PDFs

[Grete+ in prep.]



- Density var. approx. $\propto \gamma \mathrm{M_s}$
- Cooling not important



Conclusions

- Established a method to analyze energy transfer in compressible MHD
- So far: predominantly local energy transfer

- Cooling function less important than EOS
- EOS less important than compressibility

- Next: more "realistic" environments ⇒ informed model