Leibniz-Institut für Astrophysik Potsdam
AIP

BRAGINSKII-MHD IN AREPO

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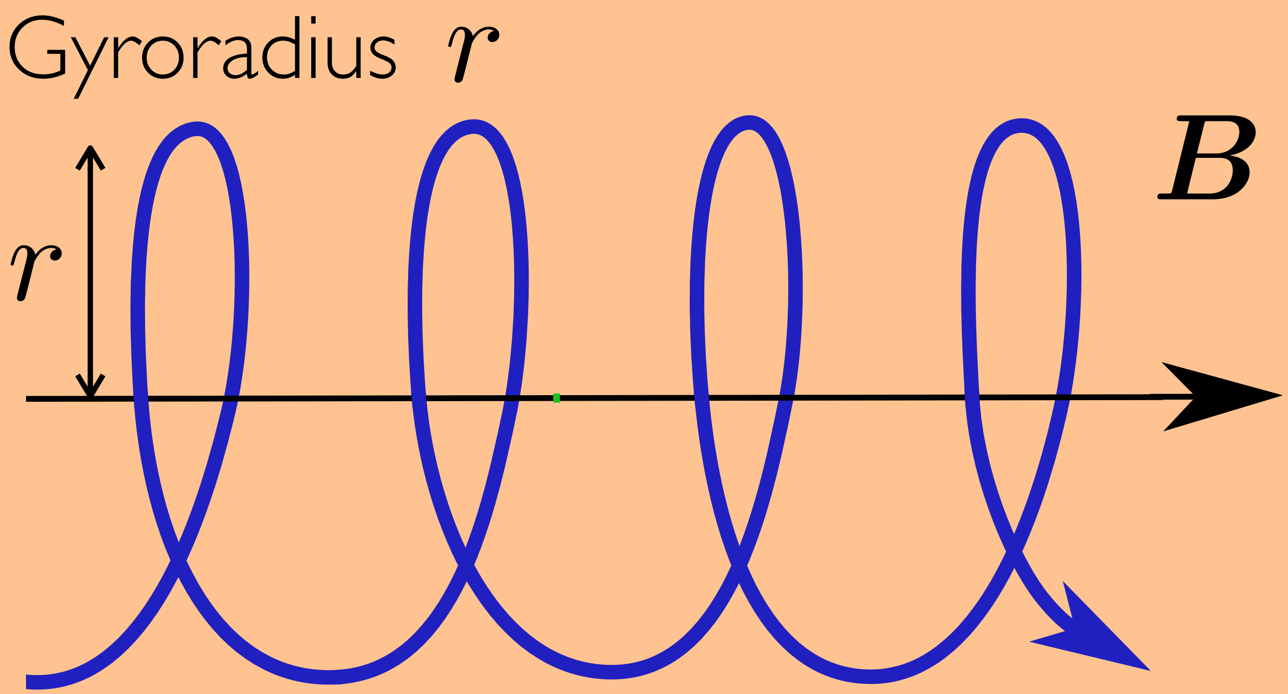
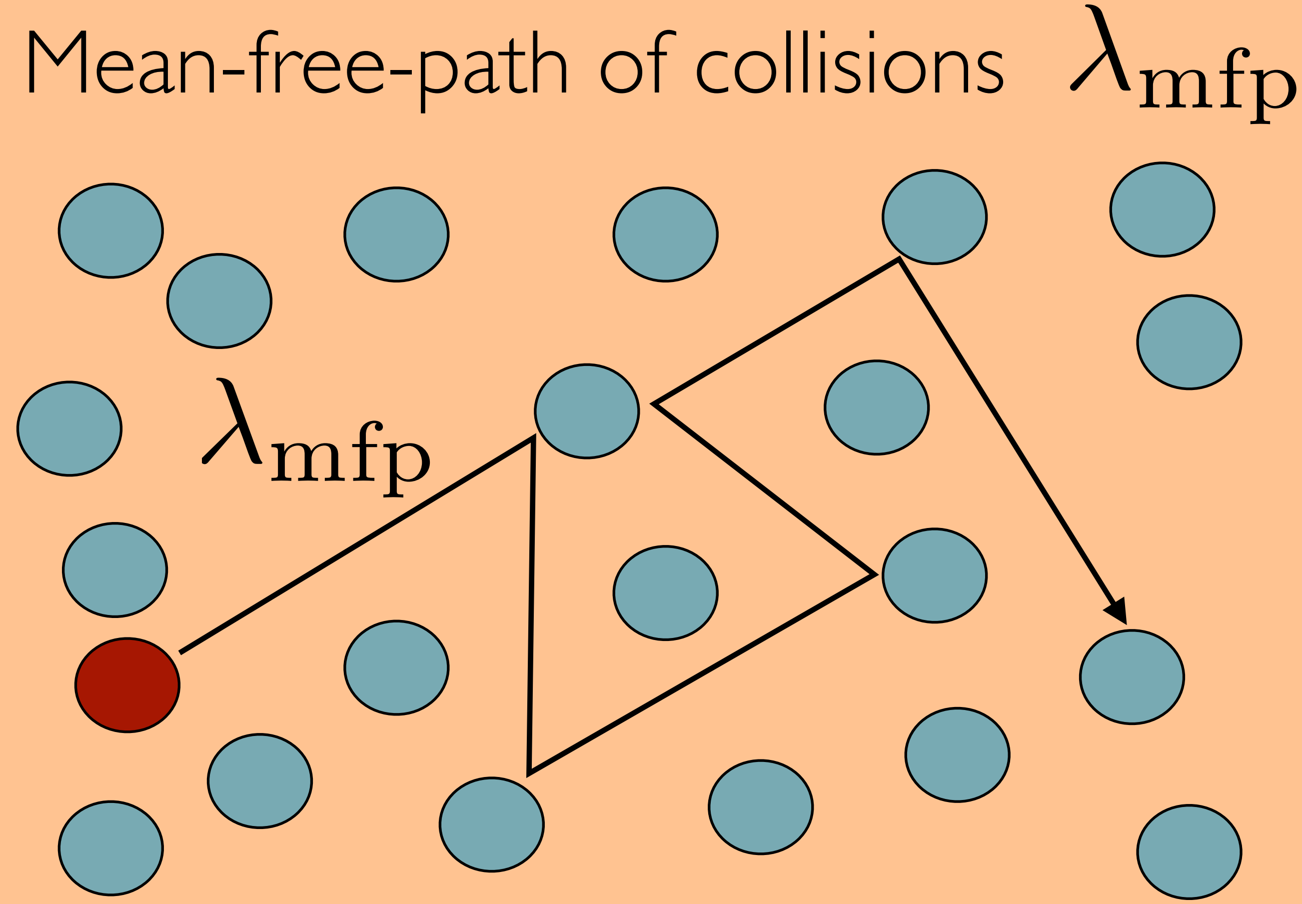
PHYSICS OF THE INTRA-CLUSTER MEDIUM: THEORY AND COMPUTATION
BUDAPEST, MARCH 4-6, 2019





Weakly collisional and magnetized

$$r \ll \lambda_{\rm mfp} \ll L$$

→ Transport of heat and momentum is along magnetic field lines.

Heat conduction

$$\boldsymbol{Q} = -\chi_{\parallel}\boldsymbol{b}(\boldsymbol{b}\cdot\nabla T)$$

Kannan+ 2016 in Arepo

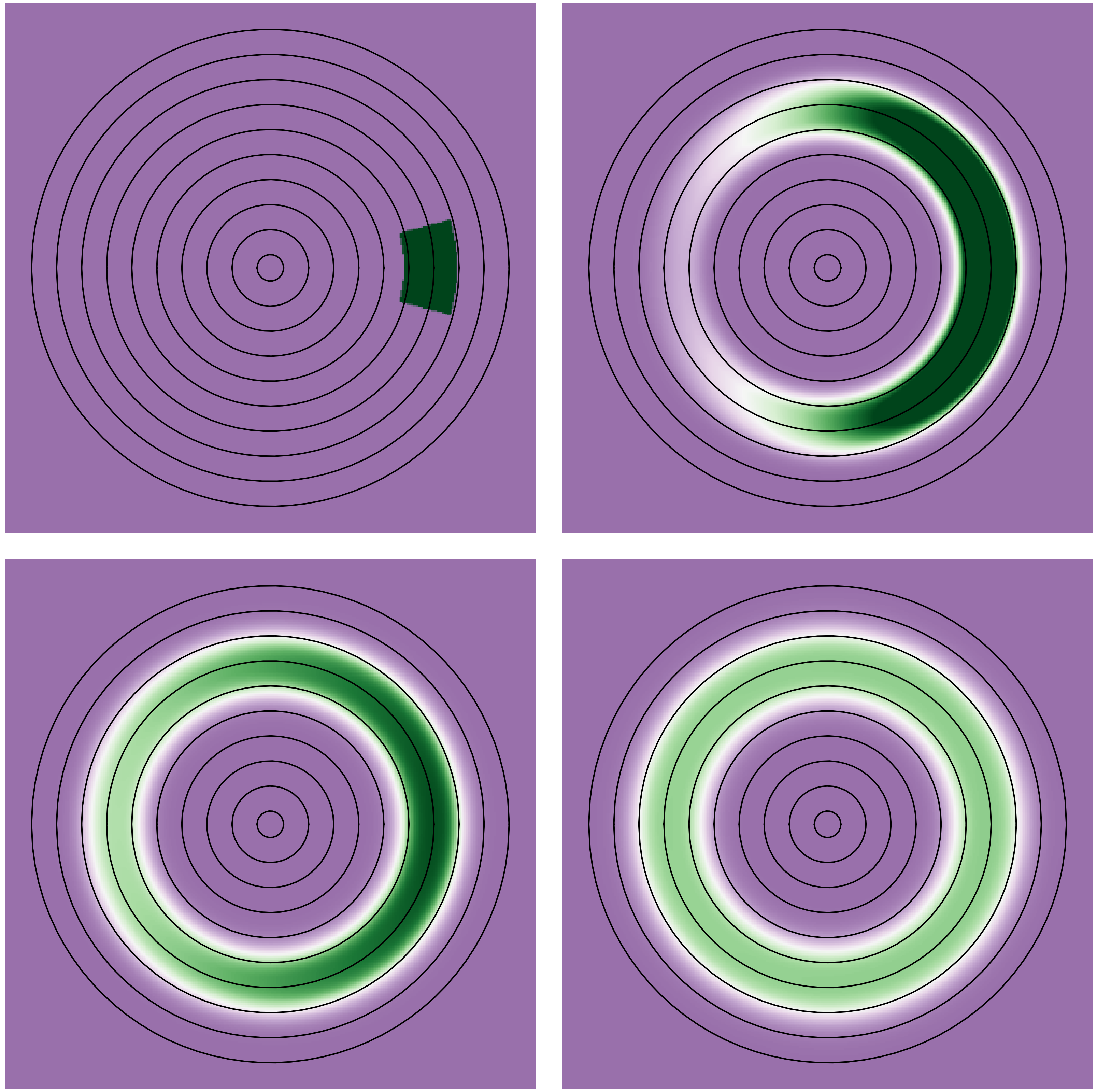
Sharma & Hammett 2007

Braginskii viscosity

$$\mathbf{\Pi} = -\Delta p\left(\boldsymbol{bb} - \frac{\mathbf{1}}{3}\right) ,$$

$$\Delta p = \rho\nu_{\parallel}(3\boldsymbol{bb} : \nabla\boldsymbol{v} - \nabla\cdot\boldsymbol{v}) .$$

ANISOTROPIC DIFFUSION



ATHENA (Stone et al. 2008)

(Parrish & Stone 2005, Sharma & Hammett 2007, Berlok & Pessah 2016a)

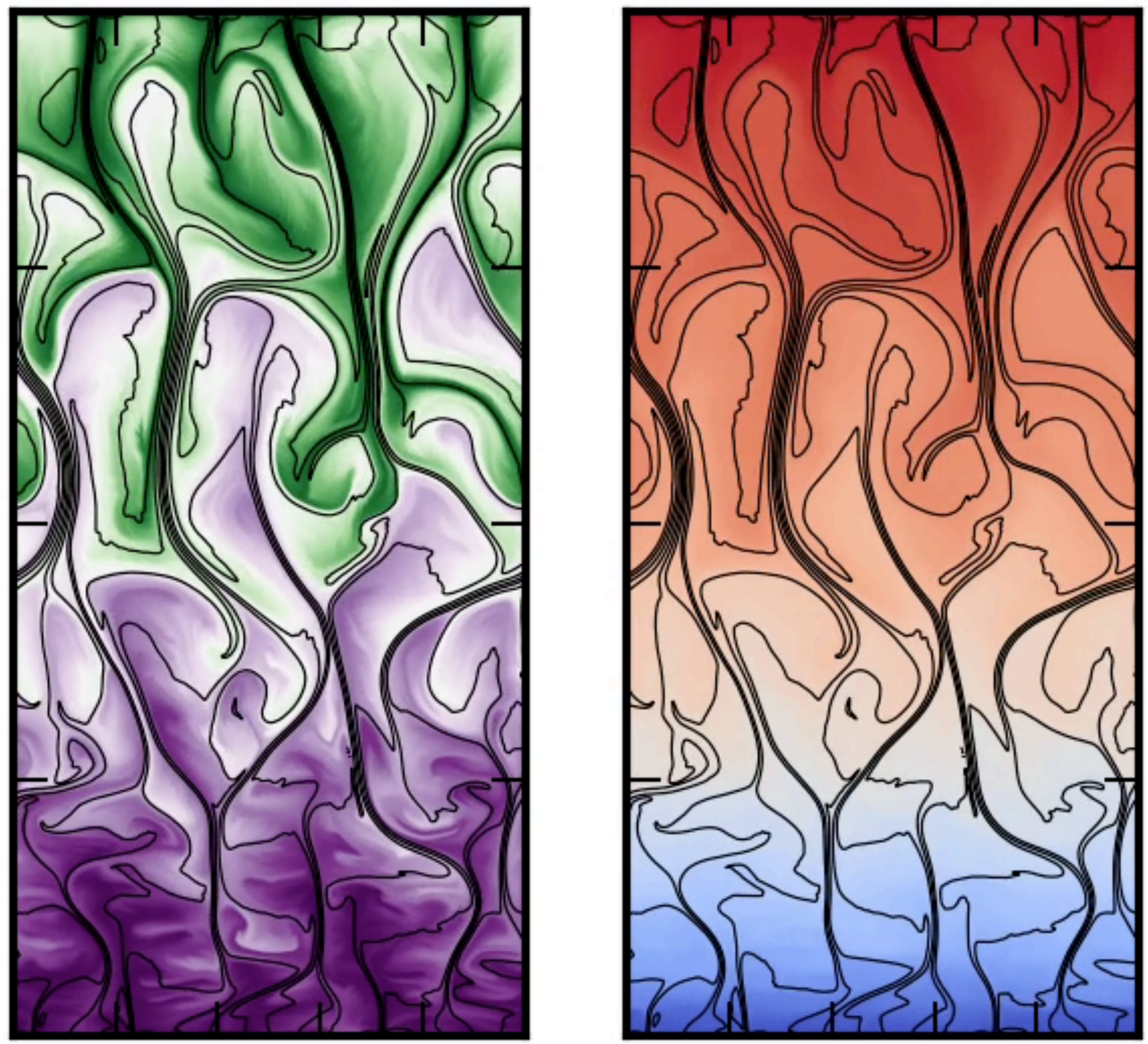
QUASI-GLOBAL SIMULATIONS

Berlok & Pessah 2016b, ApJ

$t/t_0 = 80.0$

helium

hot



$2H_0$

Peng & Nagai 2009

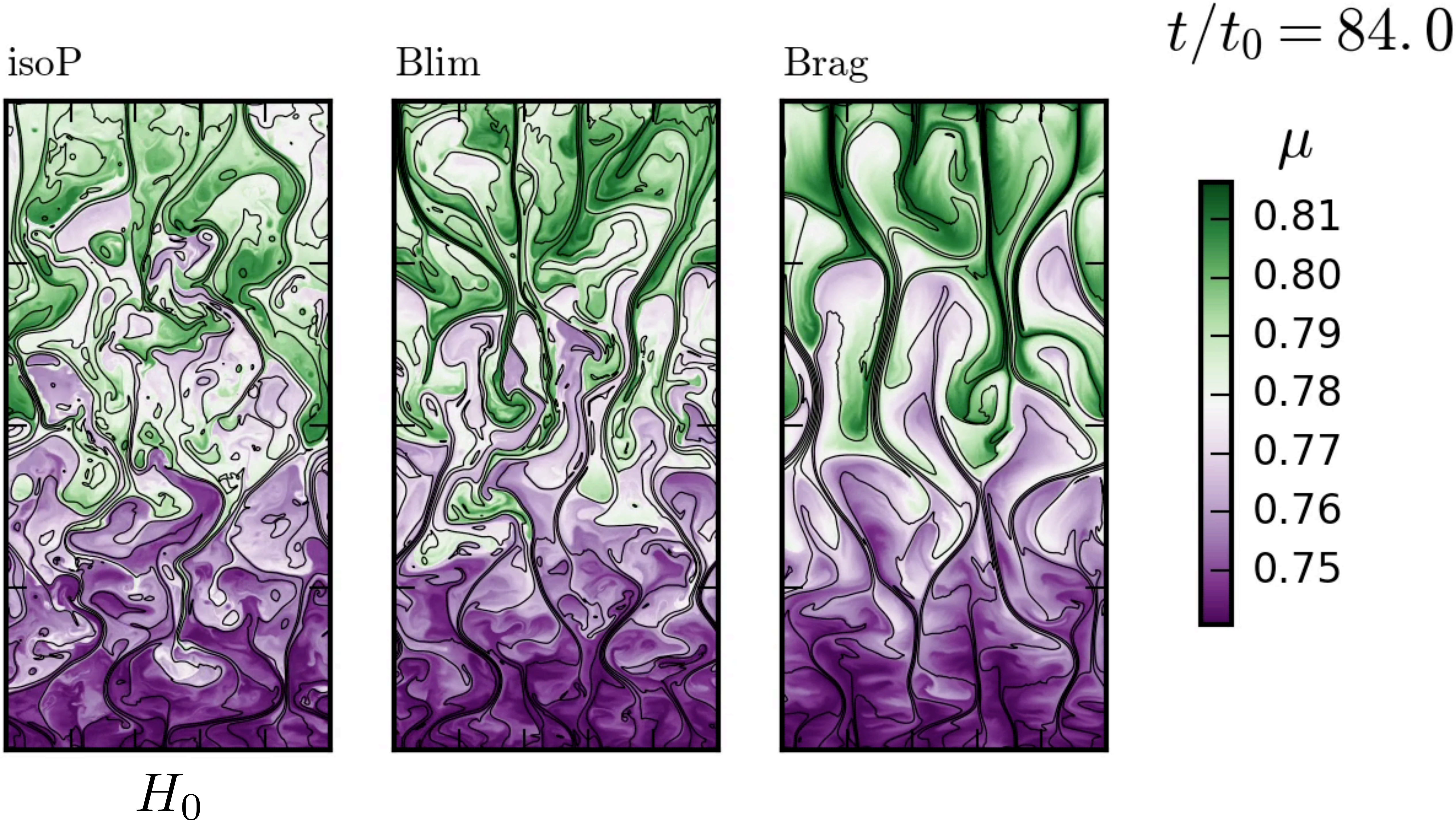
hydrogen

cold

H_0

See Balbus 2000, 2001; Parrish & Stone 2005, 2007; Quataert 2008; Parrish & Quataert 2008; Parrish et al. 2008, 2009; Bogdanovic et al. 2009; Parrish et al. 2010; Ruszkowski & Oh 2010; McCourt et al. 2011, 2012; Latter & Kunz 2012, Kunz et al. 2012; Parrish et al. 2012a,b

QUASI-GLOBAL SIMULATIONS



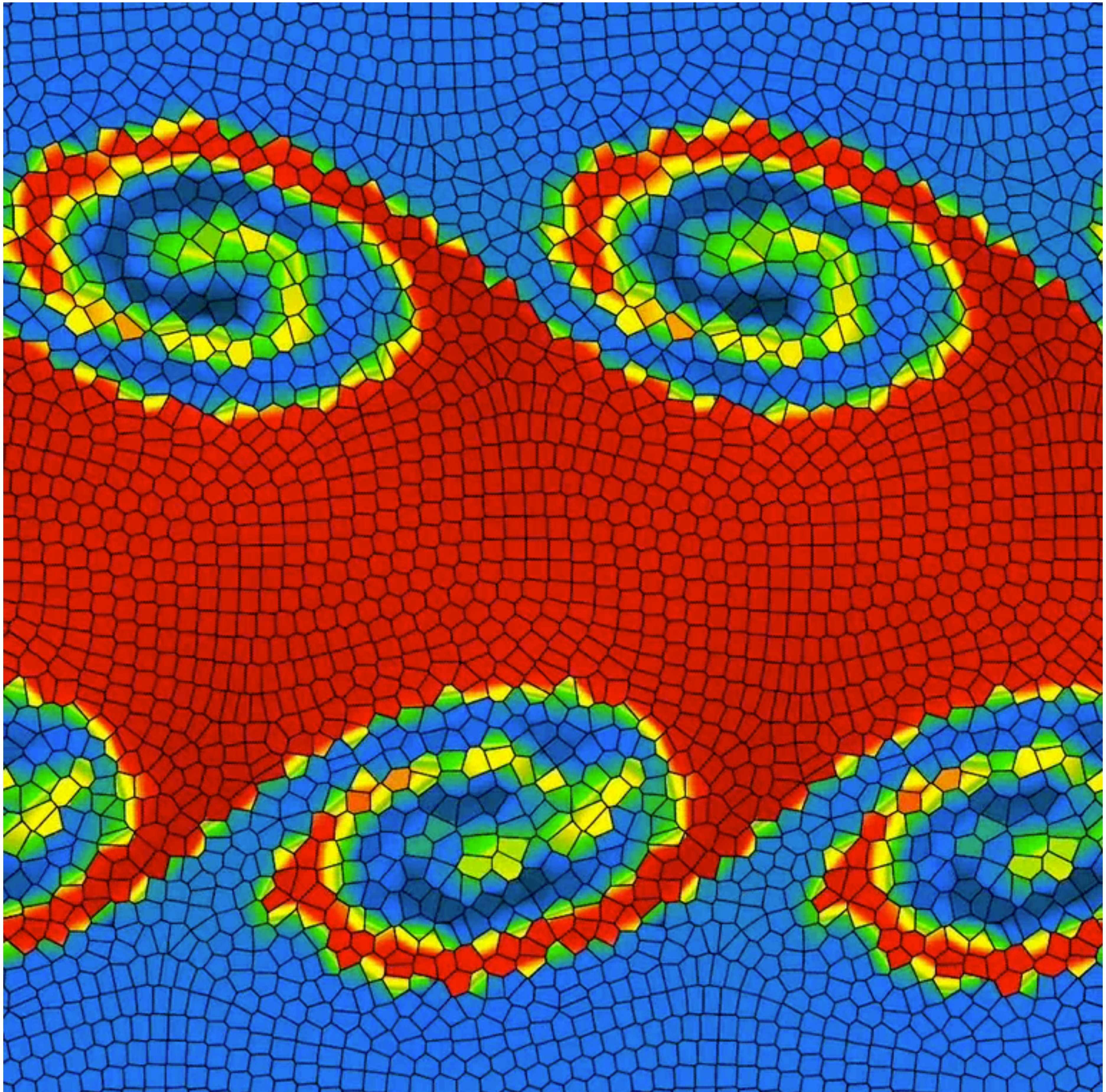
Berlok & Pessah 2016b
See also Kunz+ 2012

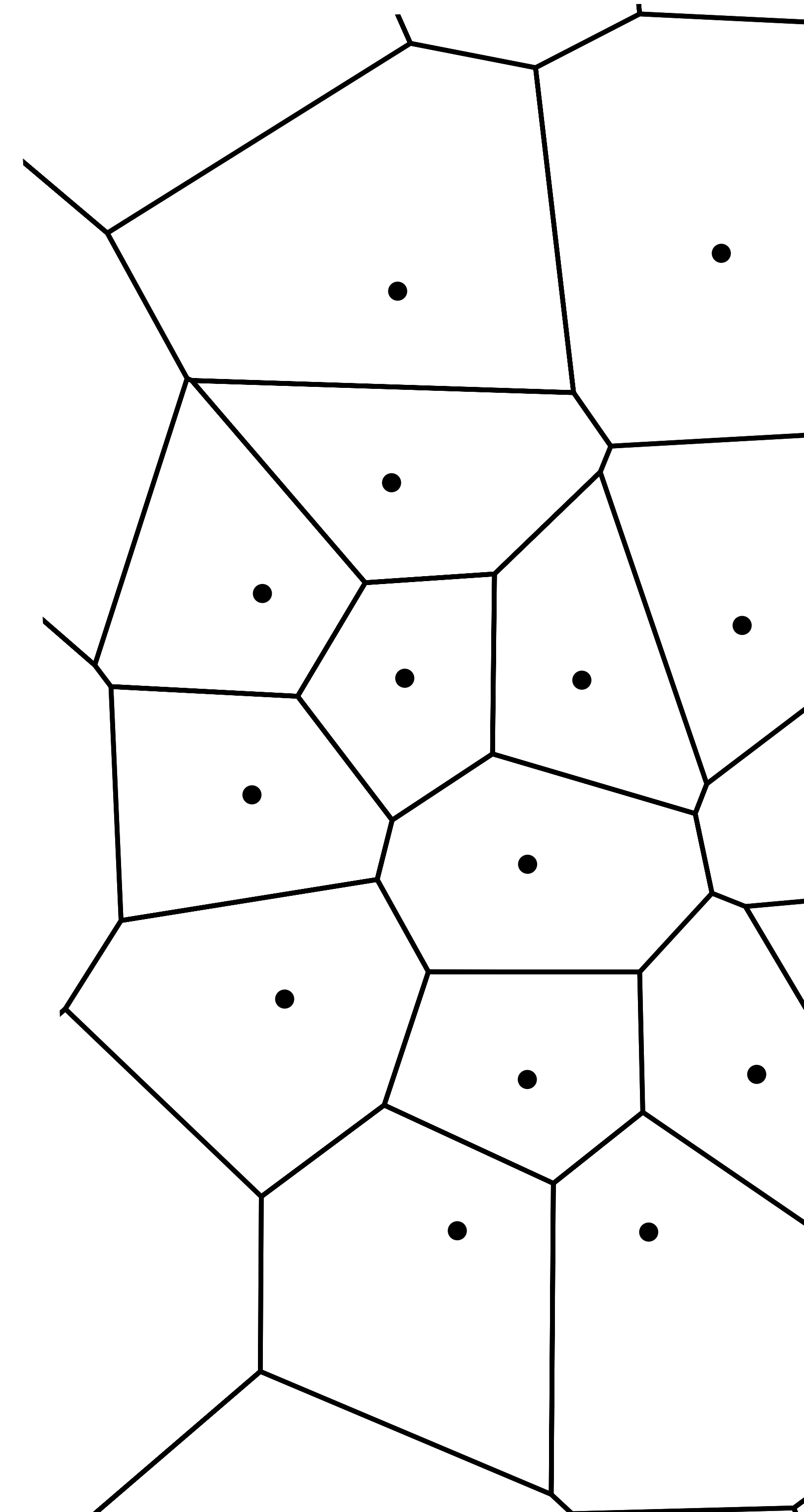
$$-\frac{B^2}{\mu_0} < \Delta p < \frac{B^2}{2\mu_0}$$

See Sharma+ 2006,
Schekochihin+ 2008, Kunz+ 2014

THE MOVING MESH CODE AREPO

Volker Springel (2010)





Pakmor+ 2011, 2013, 2016

Mocz+ 2016

BRAGINSKII VISCOSITY IN AREPO

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} = -\nabla \cdot \boldsymbol{\Pi},$$
$$\frac{\partial E}{\partial t} = -\nabla \cdot (\boldsymbol{\Pi} \cdot \boldsymbol{v}) \ ,$$

$$\boldsymbol{\Pi} = -\Delta p \left(\boldsymbol{bb} - \frac{\mathbf{1}}{3} \right) \ ,$$
$$\Delta p = \rho \nu_{\parallel} (3 \boldsymbol{bb} : \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{v}) \ .$$

SECOND ORDER ACCURATE SUPER TIMESTEPPING

$$\Delta t \leq C \frac{(\Delta x)^2}{2 d \nu_{\parallel}}$$

$$\tau = \frac{\Delta t}{4} \left(s^2 + s - 2 \right)$$

$$\frac{\partial \boldsymbol{v}}{\partial t} = L(T, \boldsymbol{v})$$
$$\frac{\partial E}{\partial t} = \nabla \cdot \boldsymbol{F}_E(T, \boldsymbol{v})$$

Velocity update

$$Y_0 = \boldsymbol{v}^n \ ,$$
$$Y_1 = Y_0 + \tilde{\mu}_1 \tau L(T^n, Y_0) \ ,$$
$$Y_j = \mu_j Y_{j-1} + \nu_j Y_{j-2} + (1 - \mu_j - \nu_j) Y_0 + \tilde{\mu}_j \tau L(T^n, Y_{j-1}) + \tilde{\gamma}_j \tau L(T^n, Y_0) \quad \text{for } 2 \leq j \leq s$$
$$\boldsymbol{v}^{n+1} = Y_s \ .$$

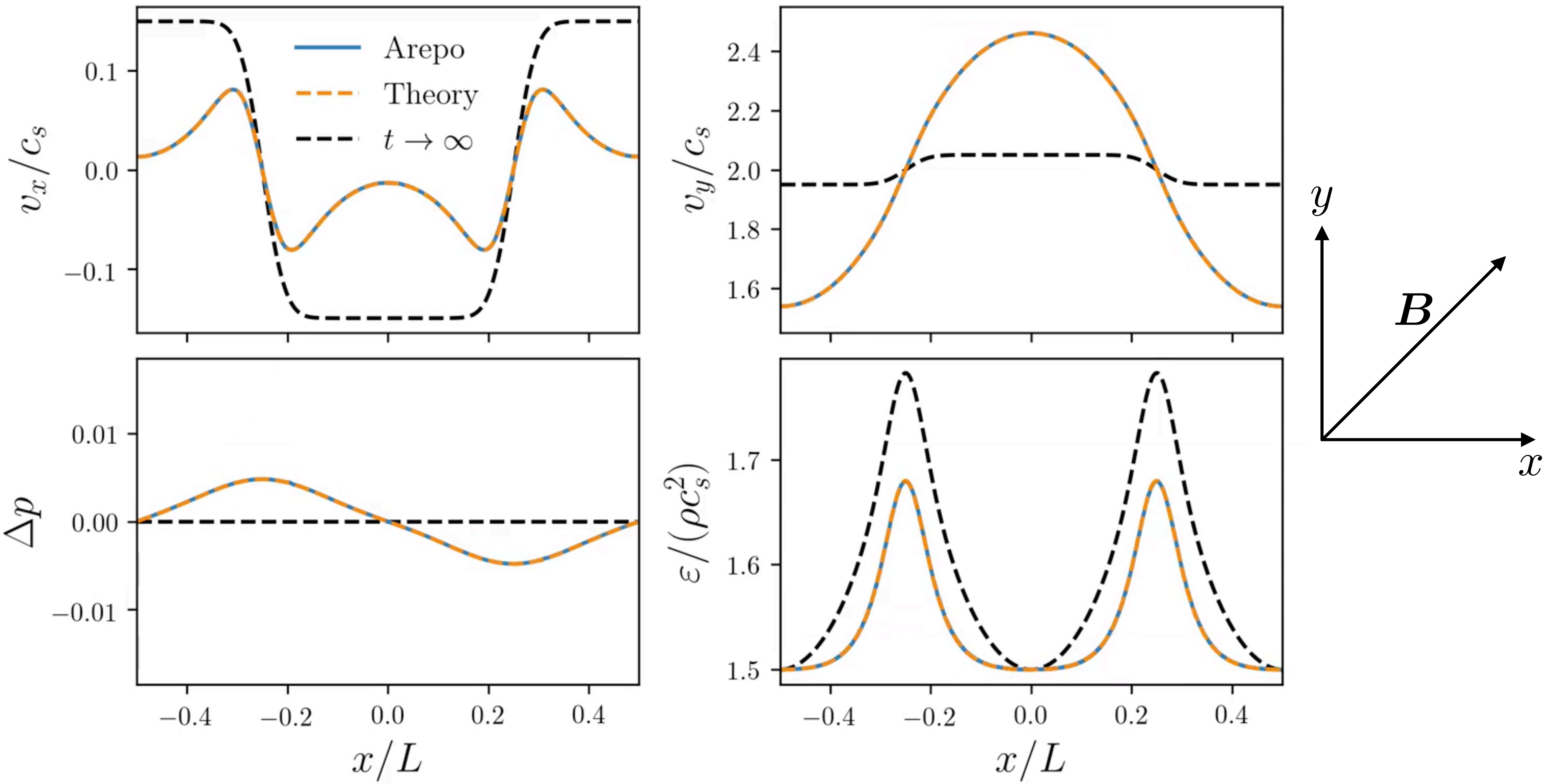
Energy update

$$E^{n+1} = \frac{\tau}{2} \left[\nabla \cdot \boldsymbol{F}_E(T^n, \boldsymbol{v}^n) + \nabla \cdot \boldsymbol{F}_E \left(T^n, \boldsymbol{v}^{n+1} \right) \right]$$

RKL2 STS theory in Meyer+ 2012. See also Vaidya+ 2017

$$v_x(x,t) = -c_s \sum_{n=0}^{\infty} \frac{3a_n}{10} \cos(k_n x)\left(1 - e^{-\gamma_n t}\right) \qquad v_y(x,t) = c_s \sum_{n=0}^{\infty} \frac{a_n}{10} \cos(k_n x)\left(1 + 9e^{-\gamma_n t}\right)$$

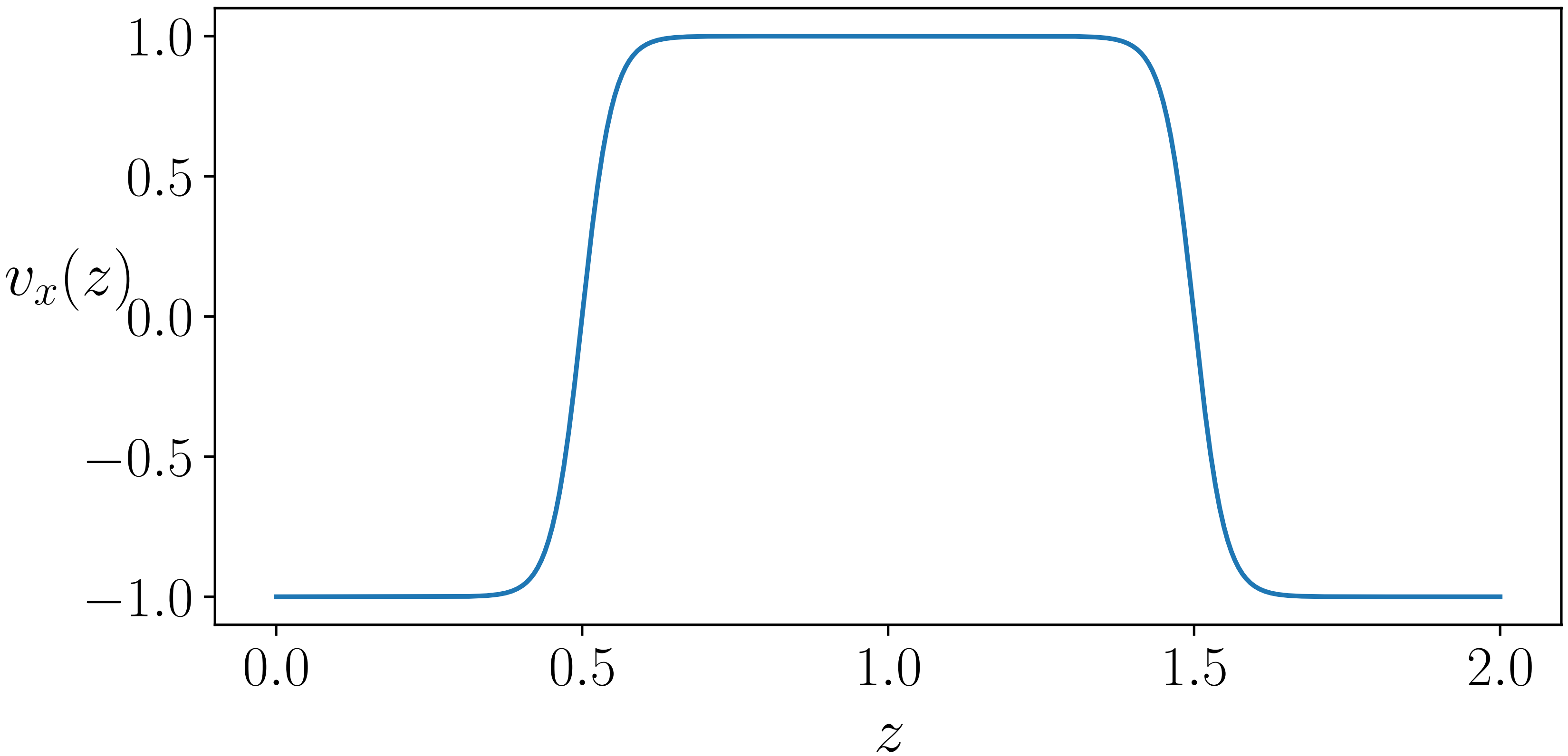
$c_s t/L = 8.5$



$$\Delta p(x,t) = -\frac{3\rho c_s \nu_\parallel}{2} \sum_{n=1}^{\infty} k_n a_n \sin(k_n x) \qquad \varepsilon(t) = \varepsilon_0 + \frac{9\rho c_s^2}{10} \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} a_n a_m \frac{\sqrt{\gamma_n \gamma_m}}{\gamma_n + \gamma_m} \times$$

$$\gamma_n = \frac{5\nu_\parallel}{6} k_n^2 \qquad \sin(k_n x)\sin(k_m x)\left(1 - e^{-(\gamma_n+\gamma_m)t}\right),$$

KELVIN-HELMHOLTZ INSTABILITY WITH BRAGINSKII VISCOSITY



Smooth equilibrium necessary for convergence of KHI.
See e.g. McNally+ 2012 and Lecoanet+ 2016.

LINEAR THEORY FOR VISCOUS KELVIN-HELMHOLTZ INSTABILITY

Berlok & Pfrommer, 2019, MNRAS
See also Suzuki+ (2013)

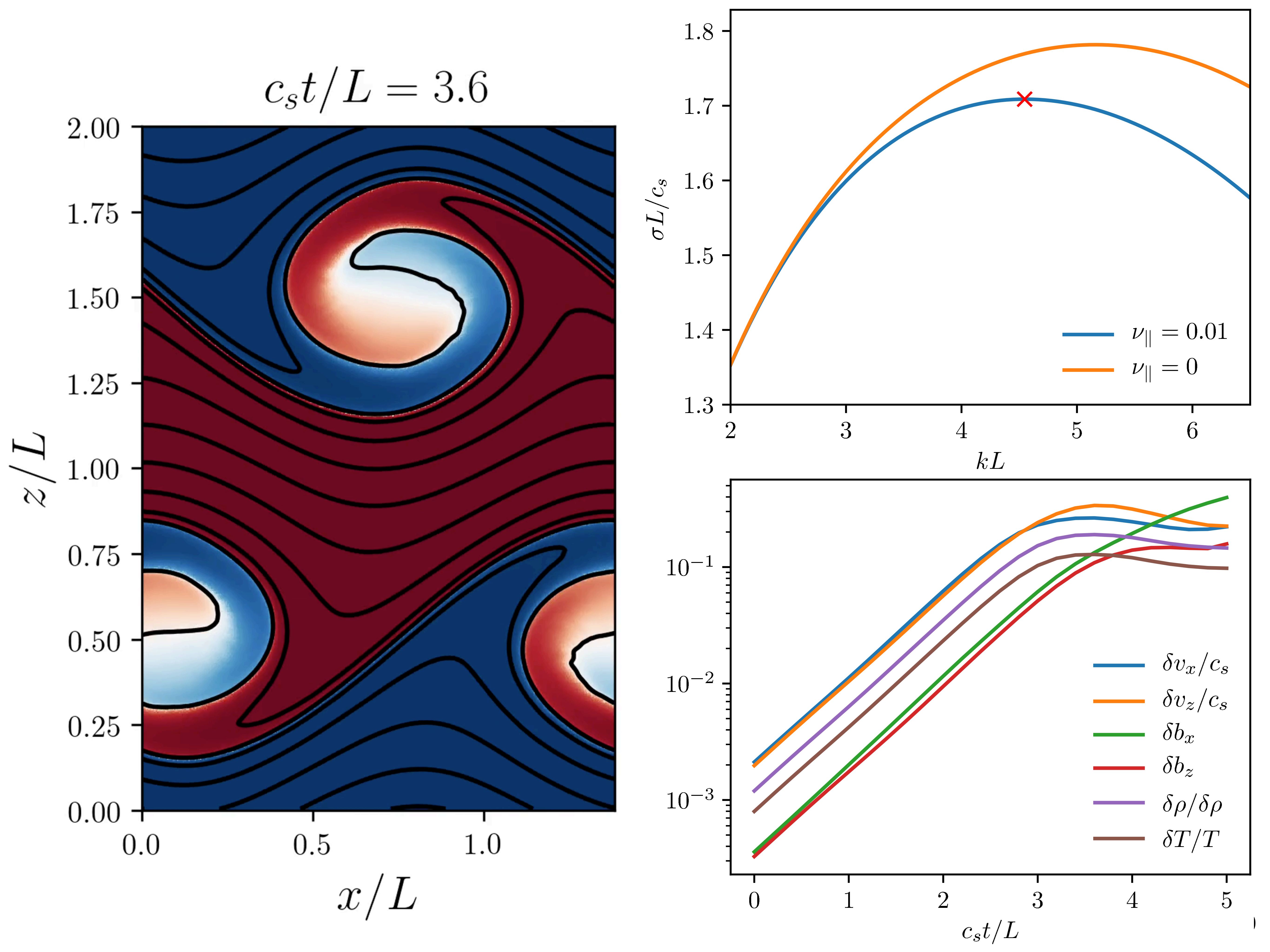
$$-i\omega\frac{\delta\rho}{\rho} = -ik\left(v_0\frac{\delta\rho}{\rho} + \delta v_x\right) - \frac{\partial\delta v_z}{\partial z}\ .$$

$$-i\omega\frac{\delta A}{B} = -ikv_0\frac{\delta A}{B} + \delta v_z\ ,$$

$$-i\omega\delta v_x = -ikv_0\delta v_x - \frac{\partial v_0}{\partial z}\delta v_z - ik\frac{\delta p}{\rho} - \nu_\parallel\left(\frac{4}{3}k^2\delta v_x + 2k^2\frac{\partial v_0}{\partial z}\frac{\delta A}{B} + ik\frac{2}{3}\frac{\partial\delta v_z}{\partial z}\right)\ ,$$

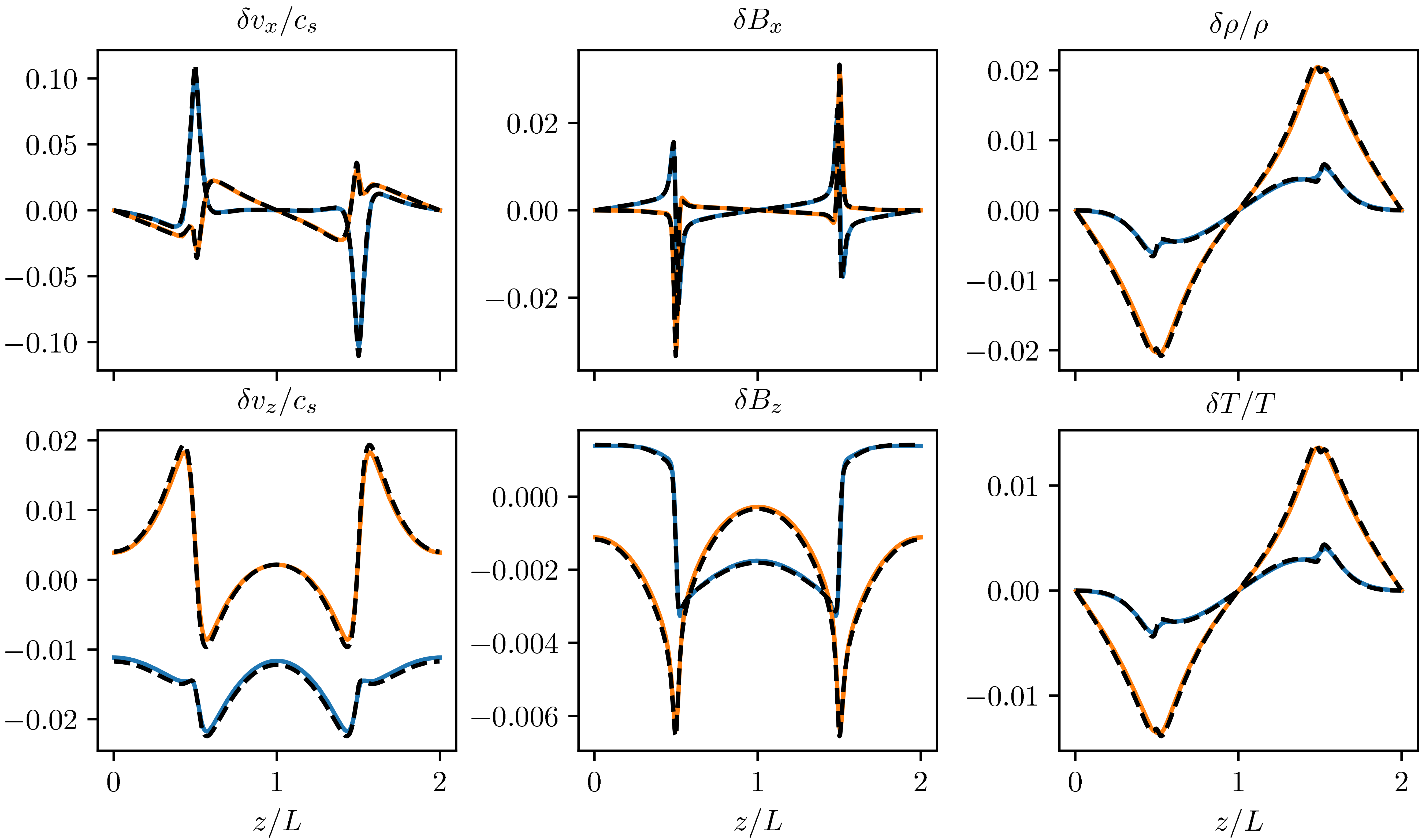
$$-i\omega\delta v_z = -ikv_0\delta v_z - \frac{1}{\rho}\frac{\partial\delta p}{\partial z} + v_a^2\left(\frac{\partial^2}{\partial z^2} - k^2\right)\frac{\delta A}{B} - \nu_\parallel\left(ik\frac{2}{3}\frac{\partial\delta v_x}{\partial z} + ik\frac{\partial^2 v_0}{\partial z^2}\frac{\delta A}{B} + ik\frac{\partial v_0}{\partial z}\frac{\partial}{\partial z}\frac{\delta A}{B} - \frac{1}{3}\frac{\partial^2\delta v_z}{\partial z^2}\right)\ ,$$

$$-i\omega\frac{\delta T}{T} = -ik\left(v_0\frac{\delta T}{T} + \frac{2}{3}\delta v_x\right) - \frac{2}{3}\frac{\partial\delta v_z}{\partial z}$$



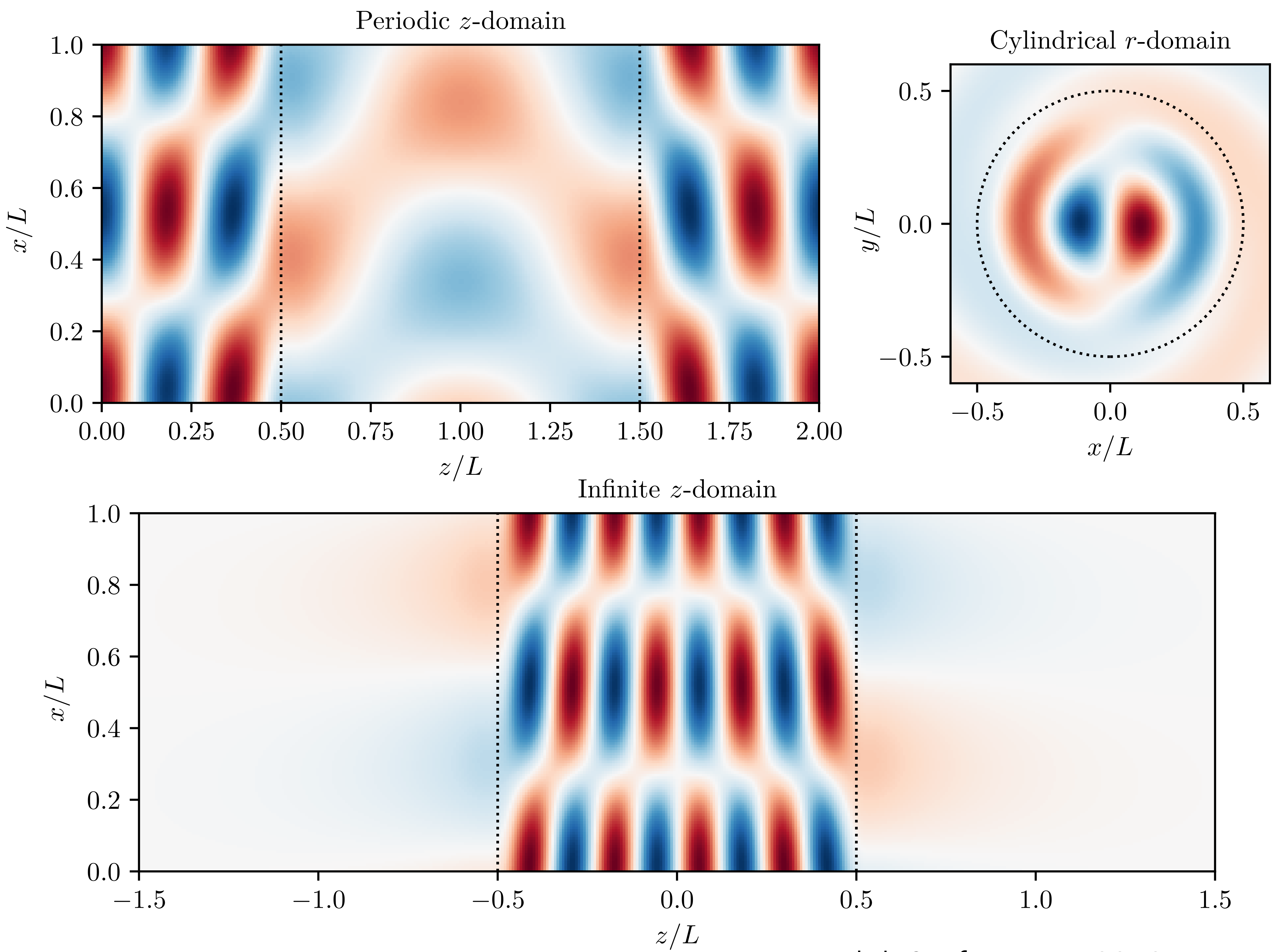
$c_s t/L = 3.6$
z/L
x/L
$\sigma L/c_s$
kL
$\nu_{\|} = 0.01$
$\nu_{\|} = 0$
$c_s t/L$
$\delta v_x/c_s$
$\delta v_z/c_s$
δb_x
δb_z
$\delta\rho/\delta\rho$
$\delta T/T$

EIGENMODES OF THE INSTABILITY



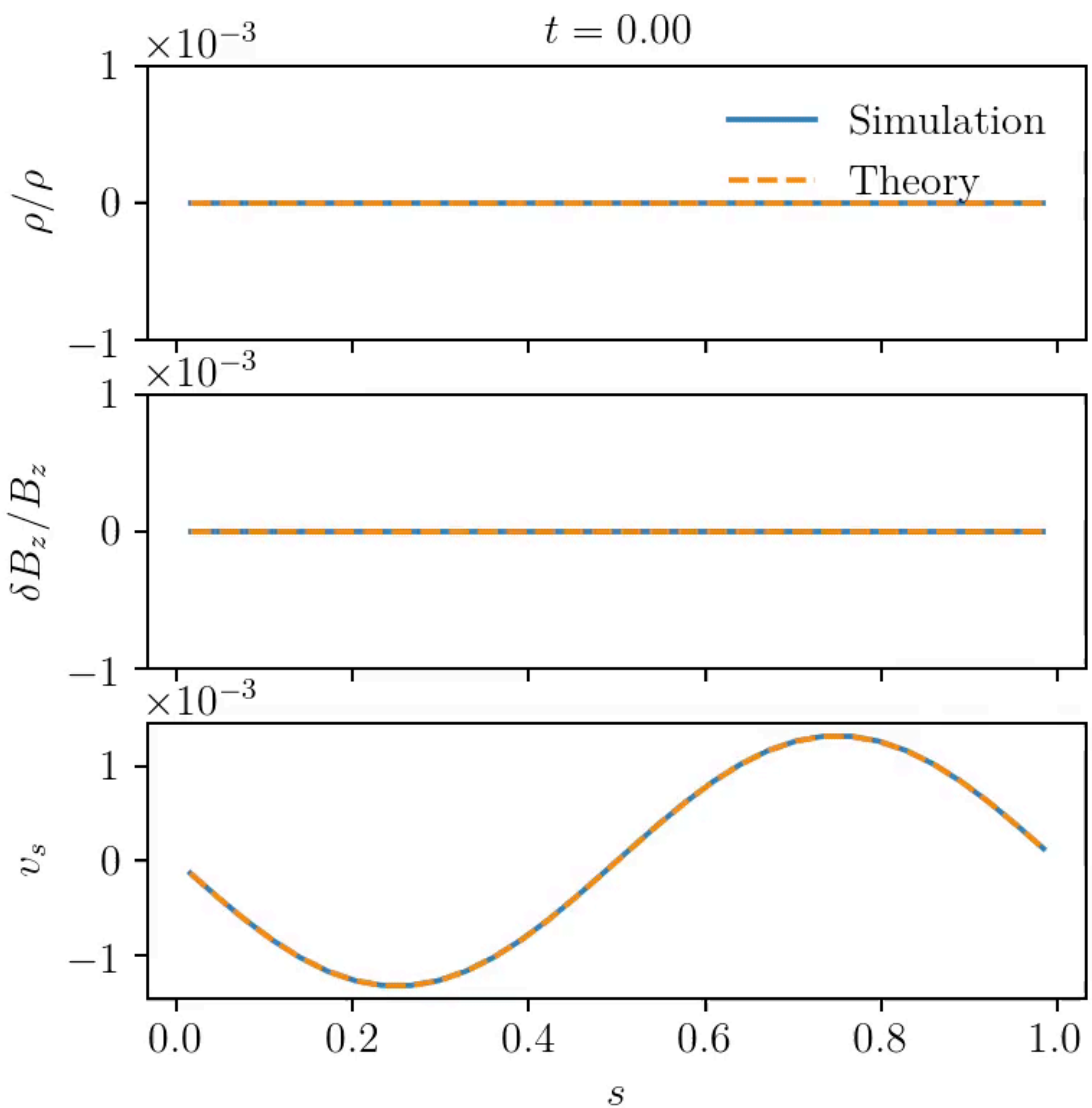
PSECAS

Pseudo-Spectral Eigenvalue Calculator with an Automated Solver
https://github.com/tberlok/psecas

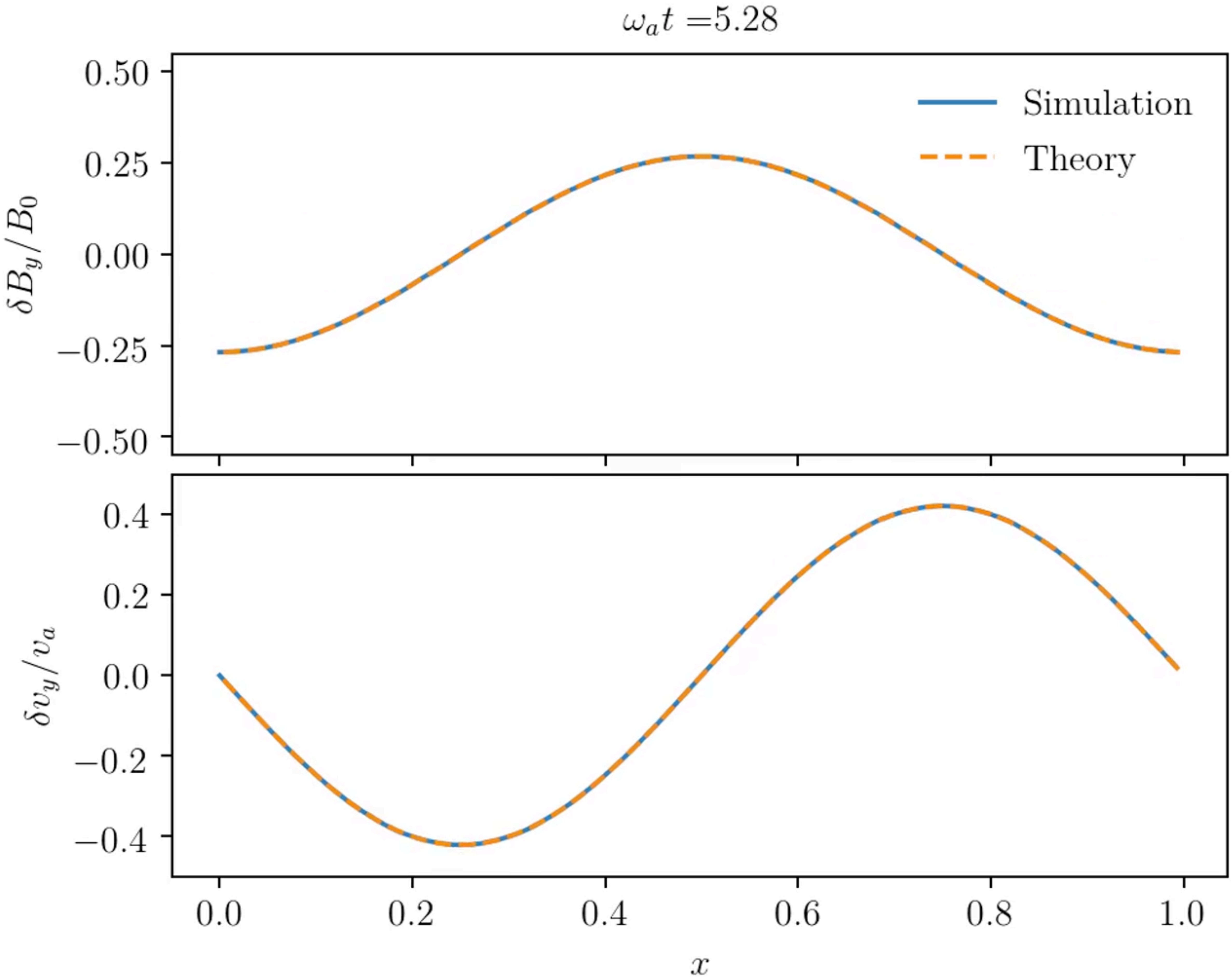


Berlok & Pfrommer, 2019, MNRAS

DECAY OF 2D MAGNETO-SONIC WAVE

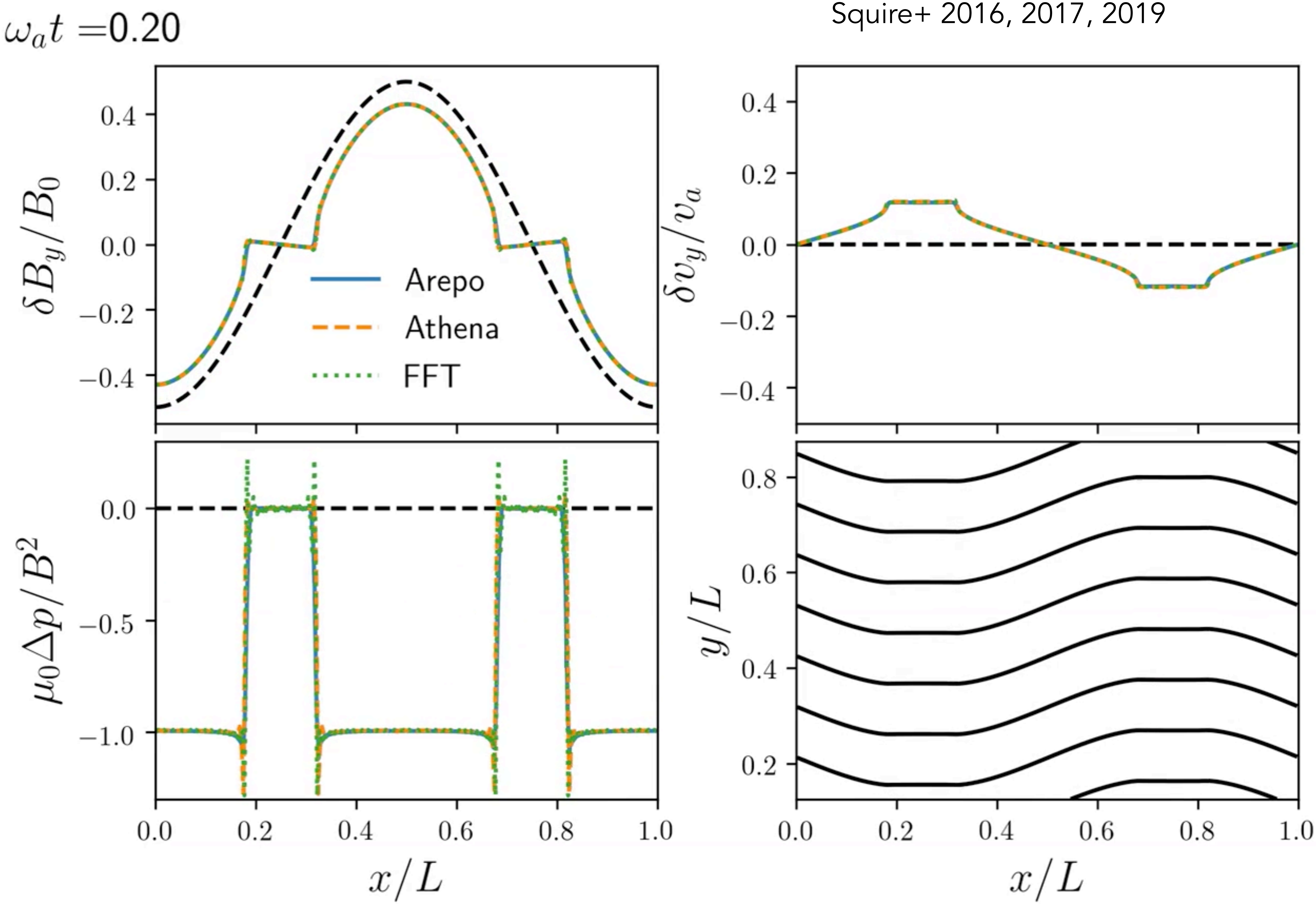


LINEARLY POLARIZED ALFVEN WAVE



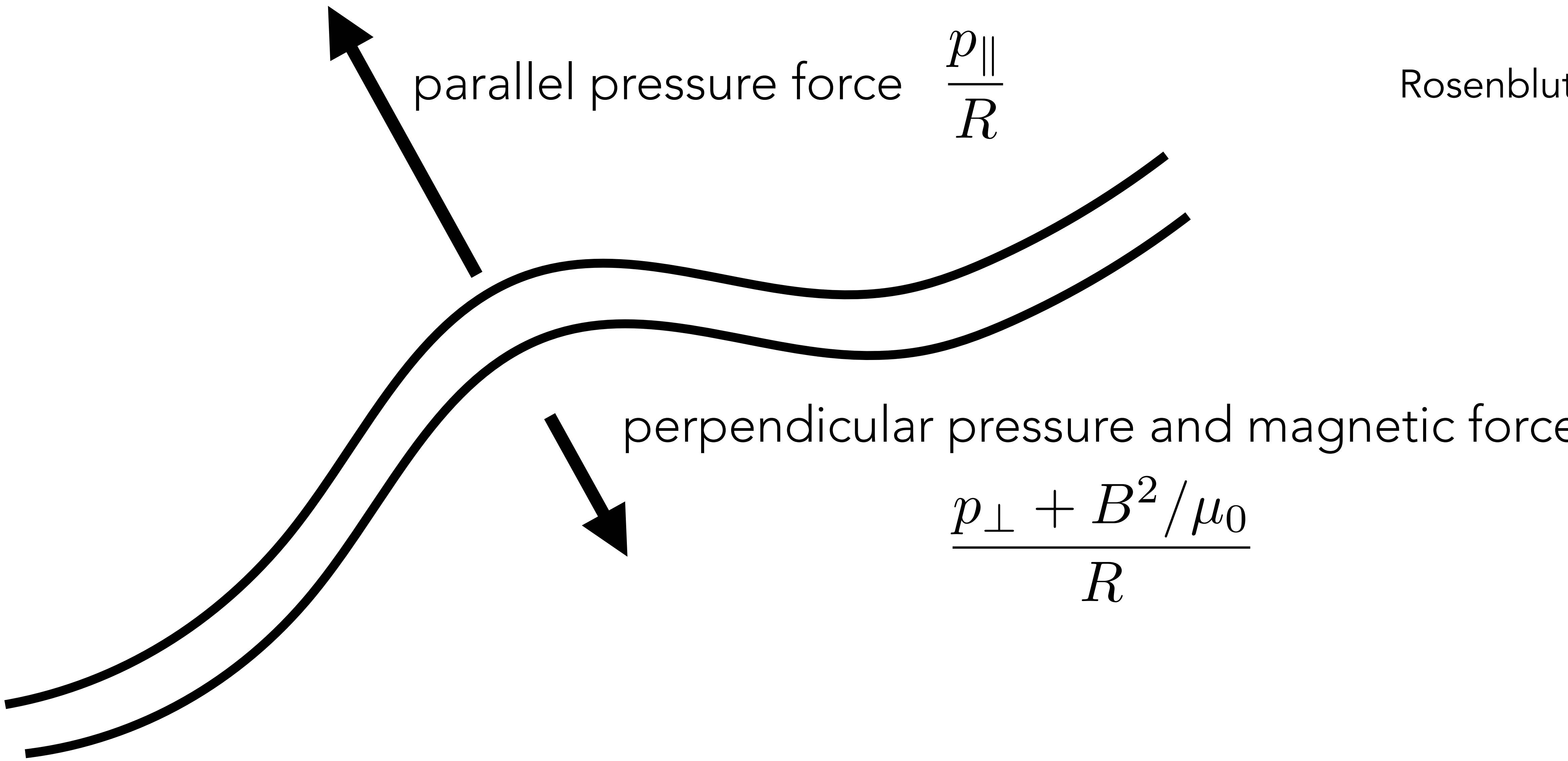
INTERRUPTION BY THE FIREHOSE INSTABILITY

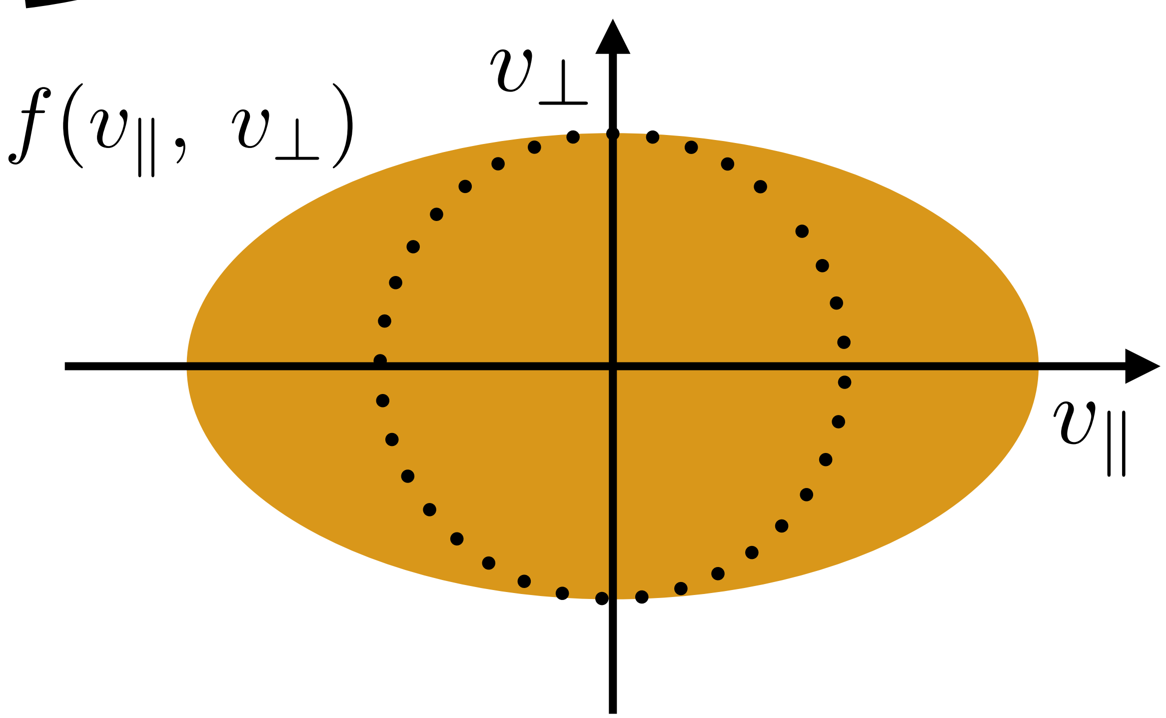
Squire+ 2016, 2017, 2019



FIREHOSE INSTABILITY

Rosenbluth 1956

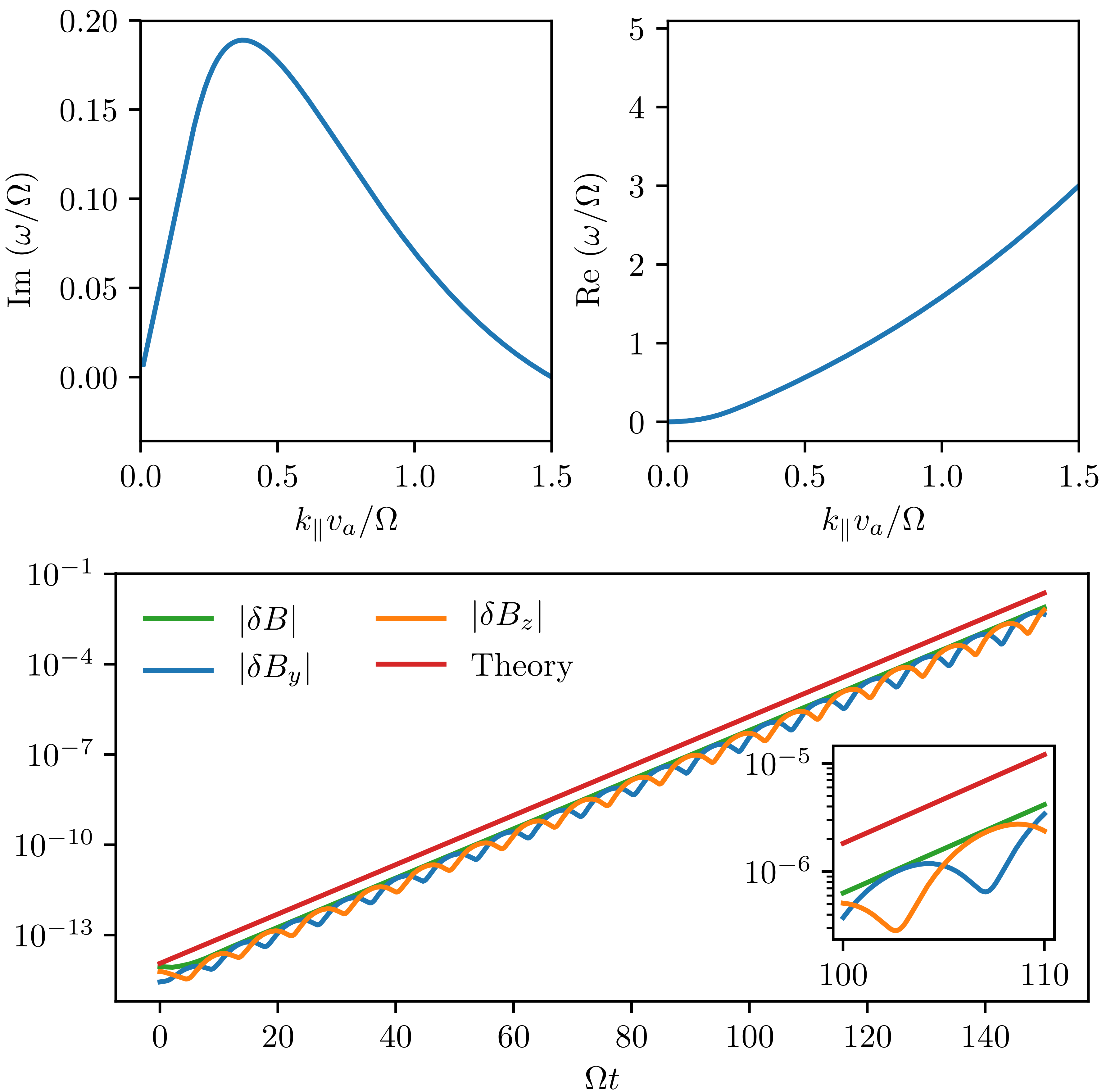




instability criterion

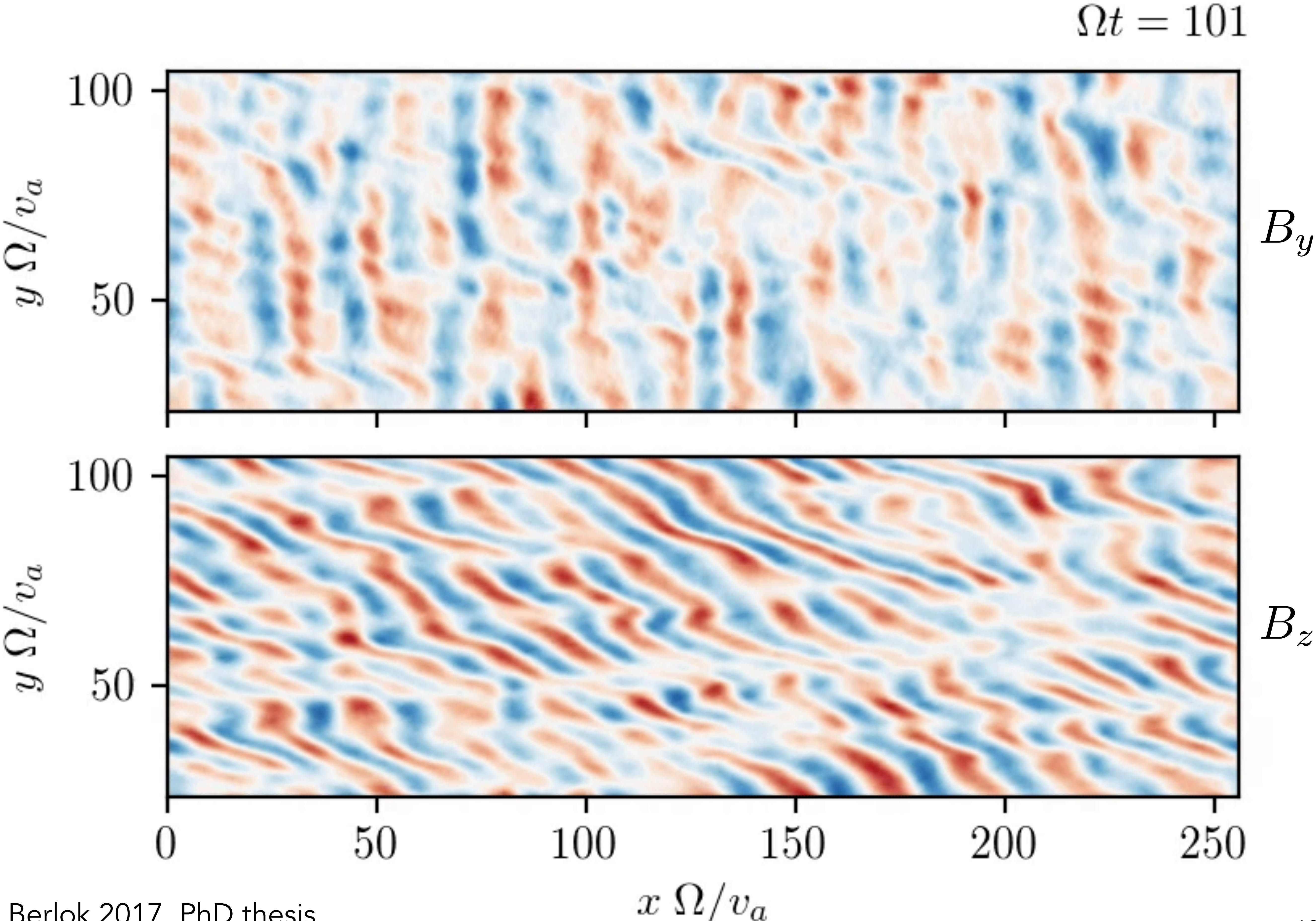
$$p_\parallel - p_\perp > \frac{B^2}{\mu_0}$$

PARALLEL FIREHOSE INSTABILITY $\beta_{\|} = 4,\ \beta_{\perp} = 1,\ T_e = 0$



Berlok 2017, PhD thesis
Advisors:
Martin Pessah, Troels Haugbølle and Tobias Heinemann
http://www.nbi.dk/~berlok/

2D FIREHOSE INSTABILITY WITH 2D-3V HYBRID-KINETIC CODE

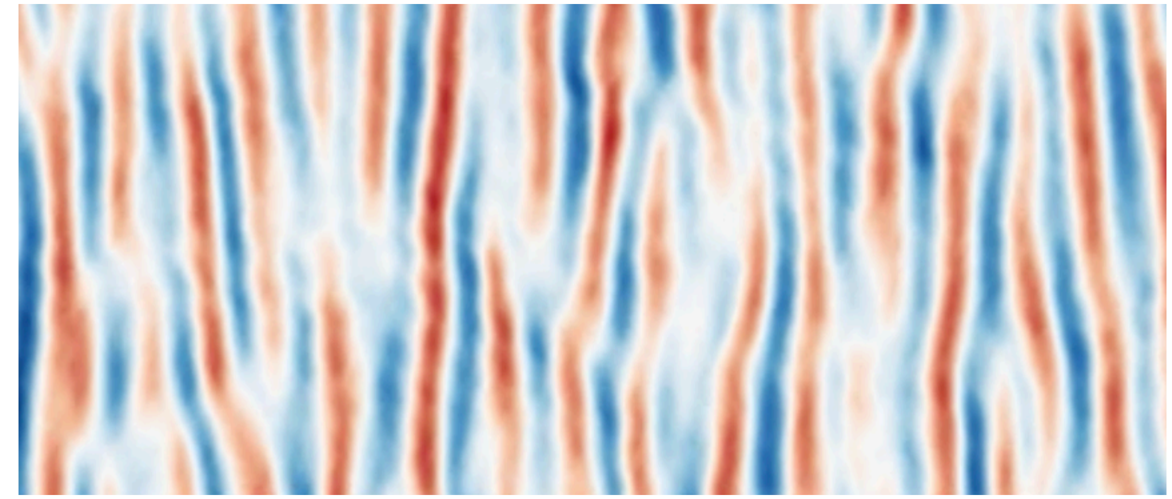


Berlok 2017, PhD thesis

SUMMARY OF MY INTERESTS

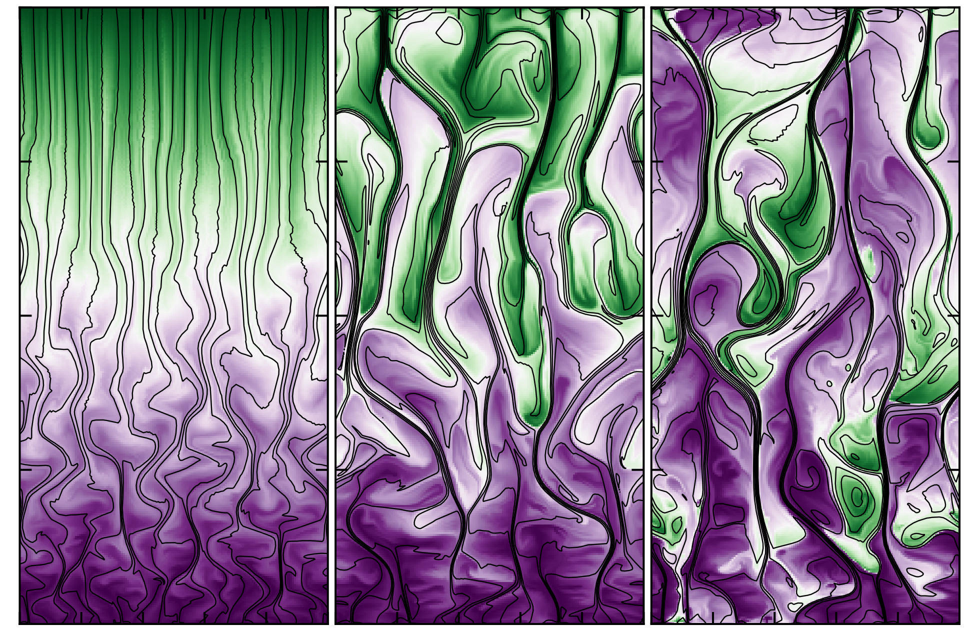
$r_i \sim 10^{-9}$ pc

Small scales
with hybrid-kinetic codes



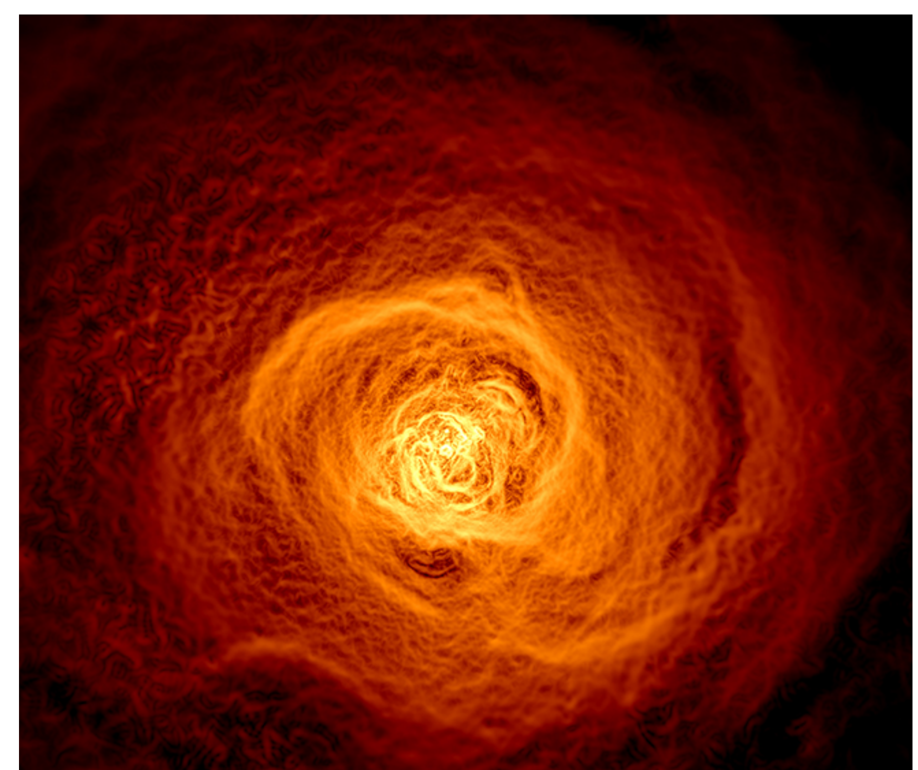
Intermediate scales with the
MHD code Athena

$H \sim 10^2$ kpc



Large scales with
Braginskii viscosity in Arepo

$L \sim$ Mpc



Chandra X-ray, Walker+ 2017

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